Topologically-Informed Atlas Learning

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PERCEPTIVE ROBOTICS AND GROUNDED REASONING SYSTEMS

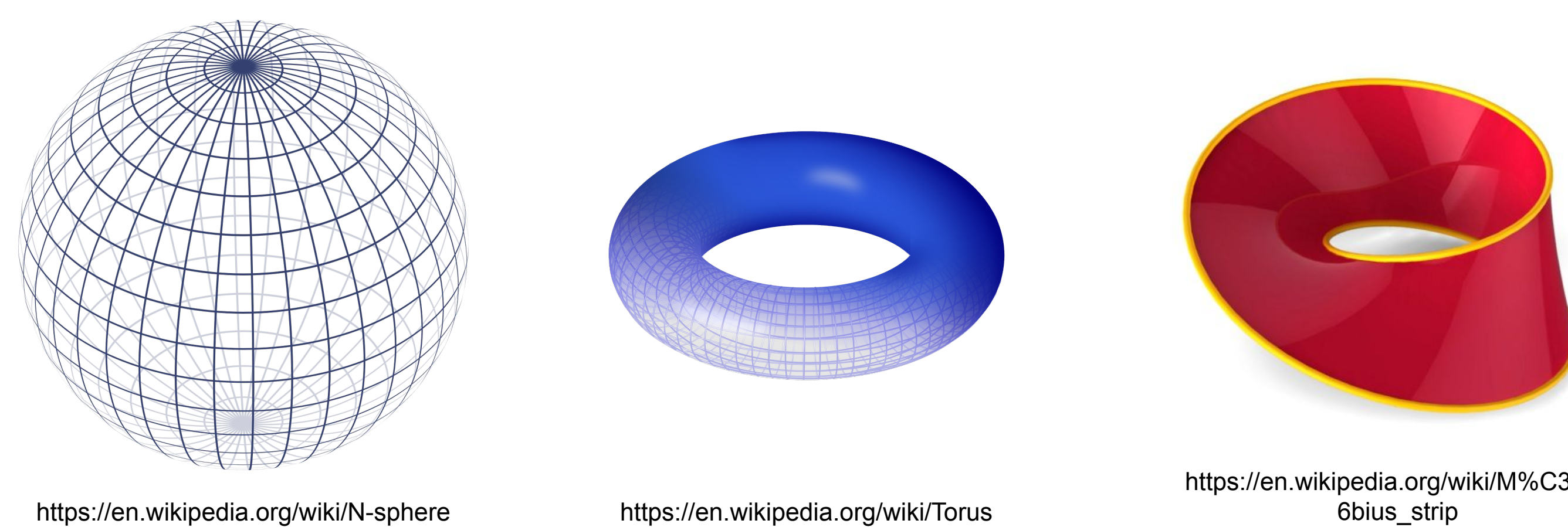ICRA 2022 IEEE International Conference on Robotics and Automation
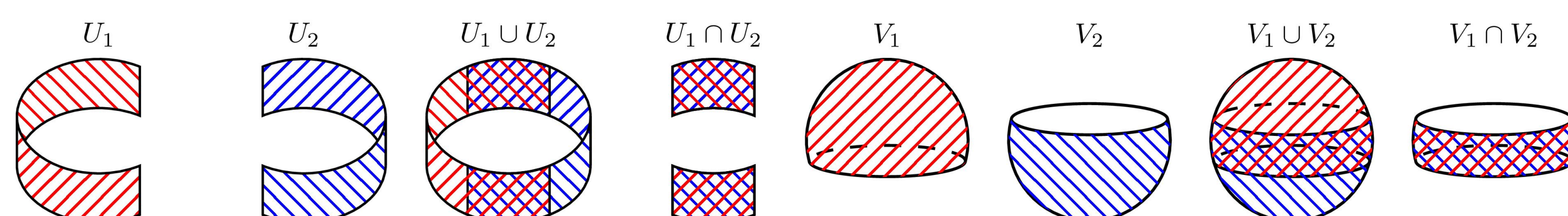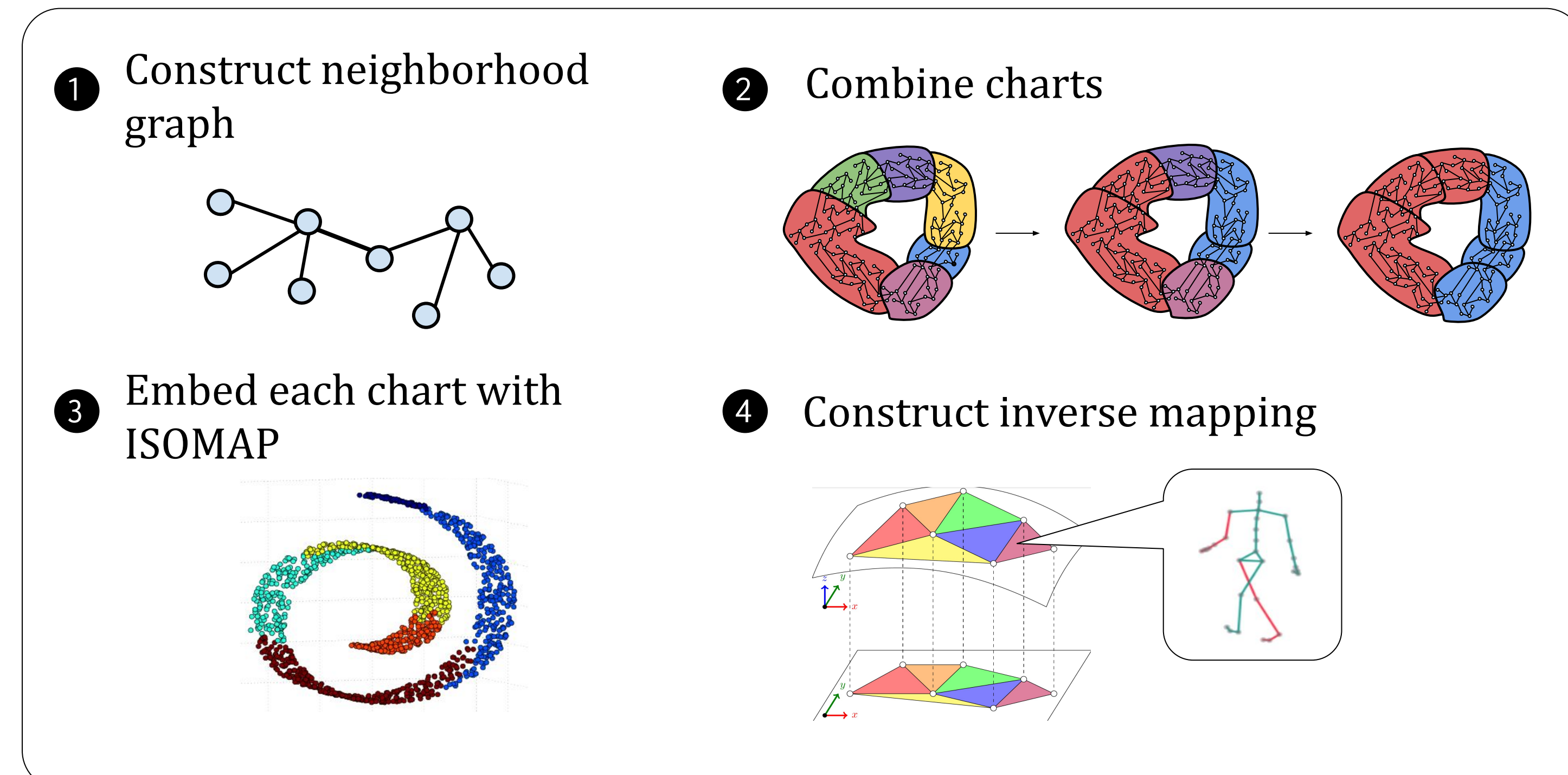
Introduction

If high dimensional data lies along some smooth manifold, it can be "unwrapped" or "flattened" to achieve dimensionality reduction. This process is called manifold learning.

Embedding produced with ISOMAP [1]
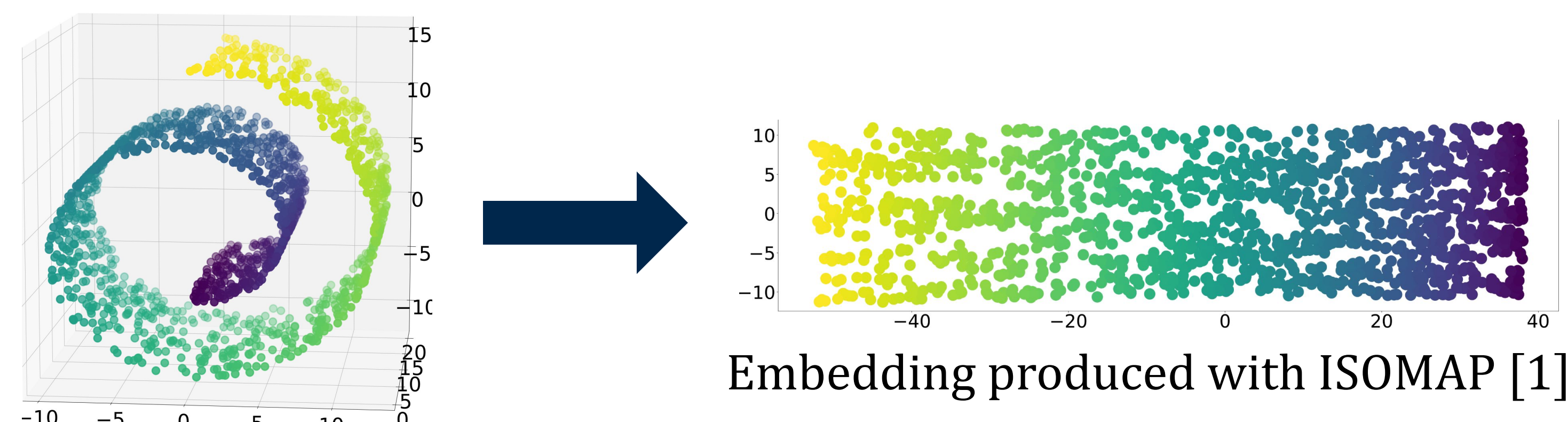
If the manifold is not topologically trivial, this approach may be impossible. For example, the sphere, torus, mobius band, SO(3), and SE(3) all cannot be flattened out. Instead, learn an atlas of coordinate charts [2][3]. We use an atlas to process human motion capture data and learn kinematic models for articulated objects [4].


https://en.wikipedia.org/wiki/N-sphere
https://en.wikipedia.org/wiki/Torus
https://en.wikipedia.org/wiki/M%C3%B6bius_strip

Methodology

1. Construct neighborhood graph
2. Combine charts
3. Embed each chart with ISOMAP
4. Construct inverse mapping

U_1 U_2 $U_1 \cup U_2$ $U_1 \cap U_2$ V_1 V_2 $V_1 \cup V_2$ $V_1 \cap V_2$

Exemplar cases where two charts cannot be combined without introducing a topological hole. $U_1 \cap U_2$ is disconnected, and $V_1 \cap V_2$ contains a hole. Holes are identified by searching for large atomic cycles [5].

Key idea: Estimating data topology improves manifold learning
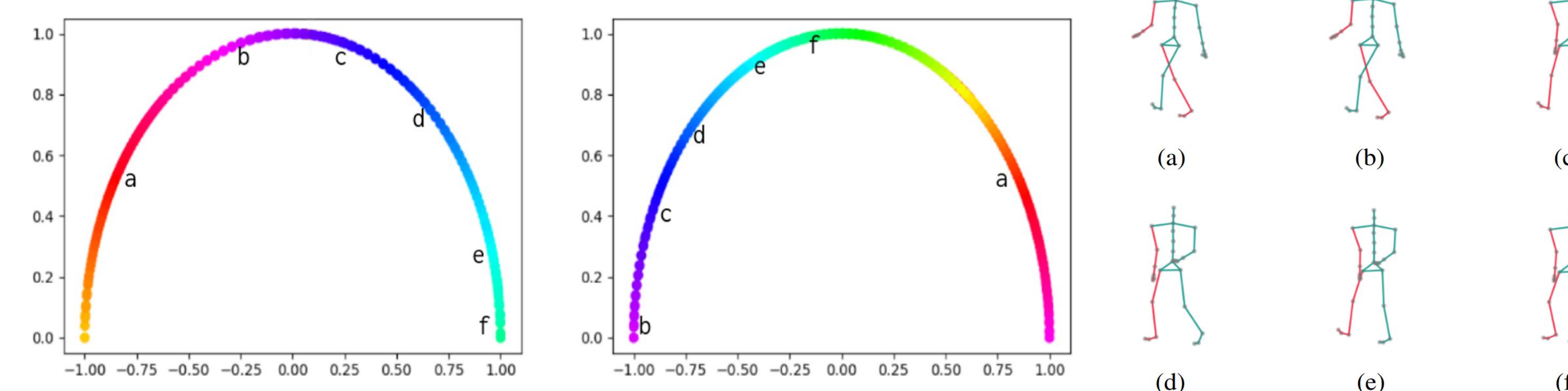
Results
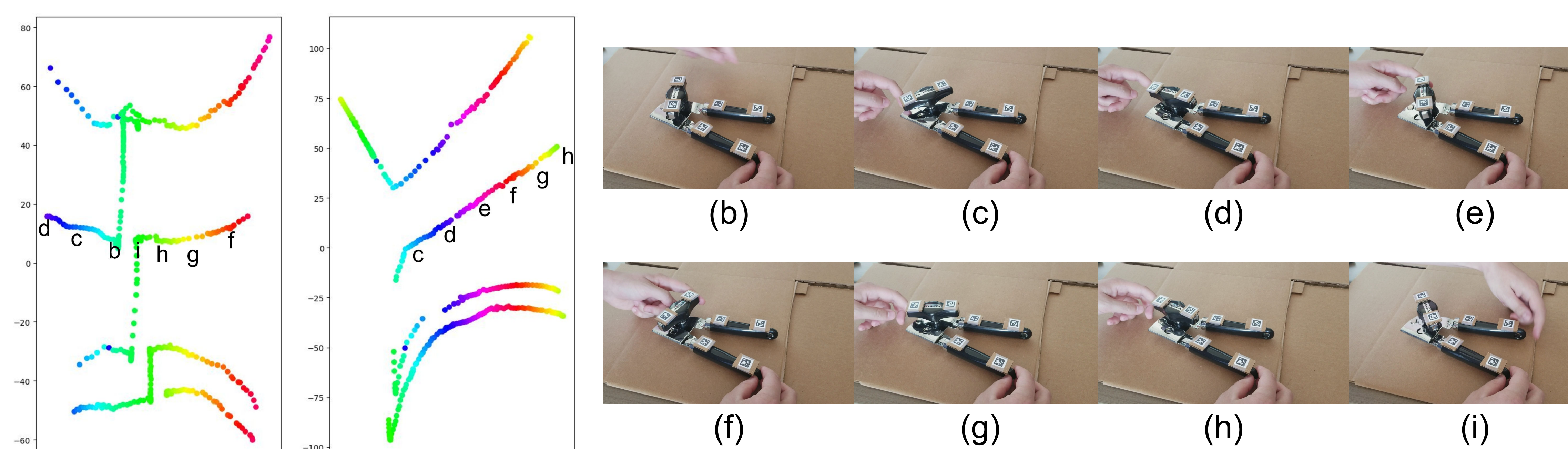
Learned kinematic model for a can opener (left), annotated with demonstrative object configurations (b)-(i).
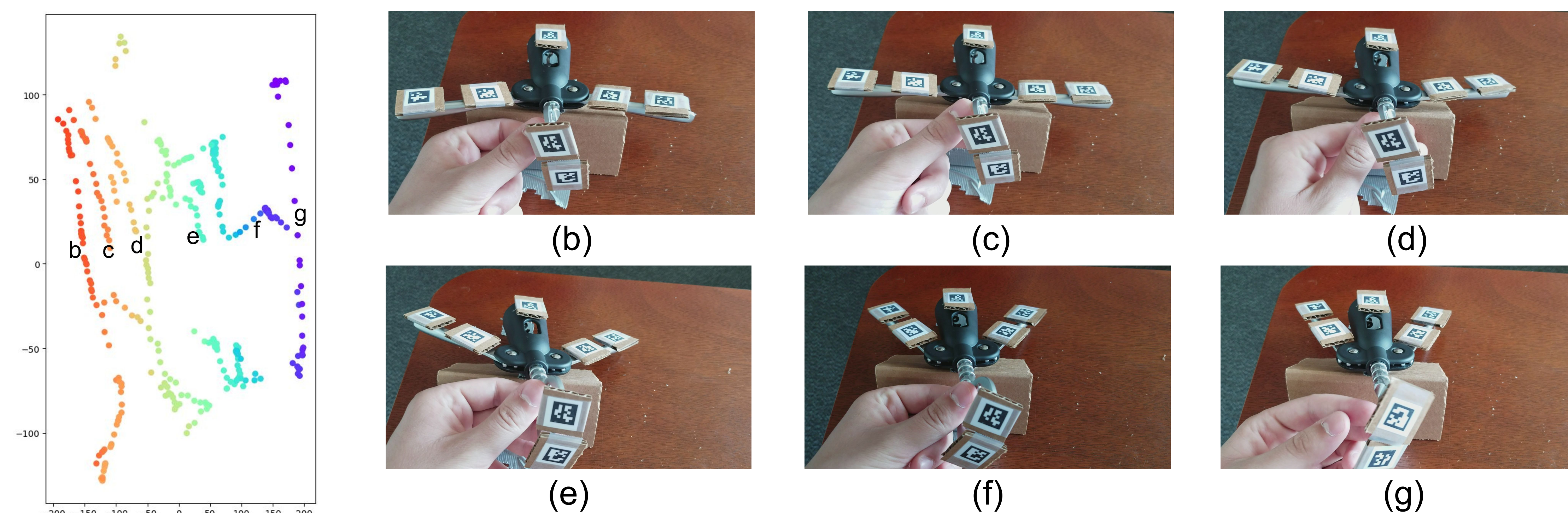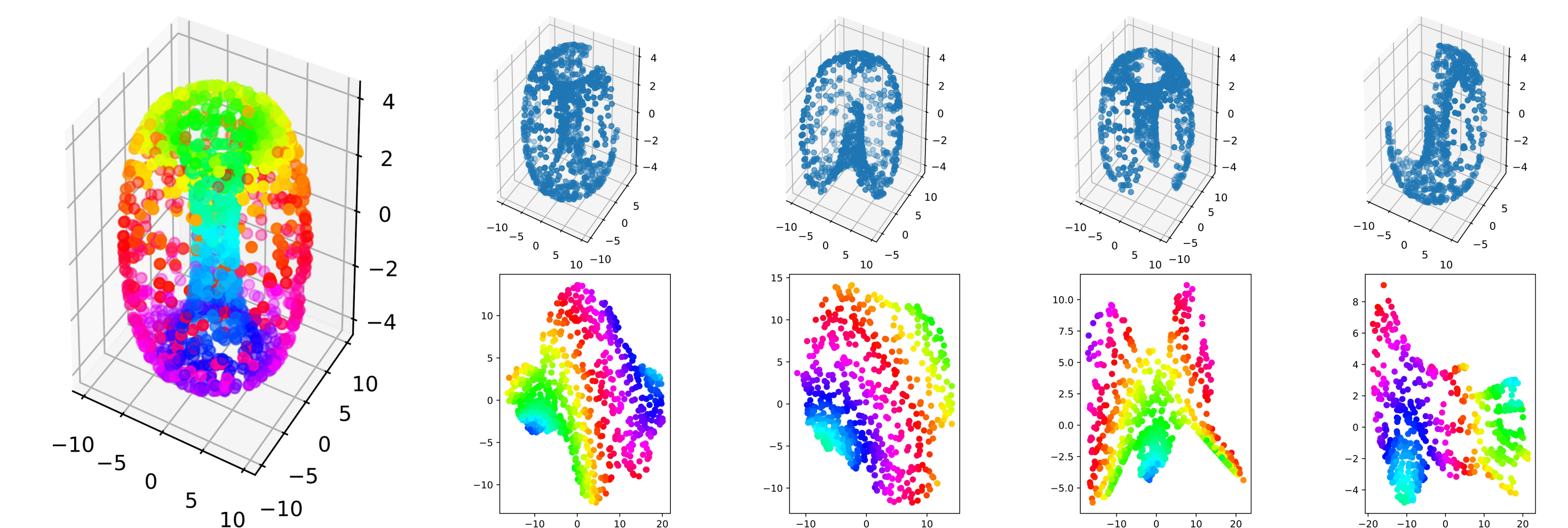

Learned kinematic model for a corkscrew bottle opener (left), annotated with demonstrative object configurations (b)-(g).

Learned embedding of motion capture data of a human walking, annotated with the embedding location of demonstrative poses.

Results (Continued)

Atlas learning for data sampled from a torus (left). Our method constructs four charts, and we show each chart domain with its corresponding embedding.

Embedding Trustworthiness							
Experiment	ISOMAP	Autoencoder (4 Charts) Worst	Autoencoder (4 Charts) Mean	Autoencoder (15 Charts) Worst	Autoencoder (15 Charts) Mean	Our Atlas Learning Worst	Our Atlas Learning Mean
Sphere	0.844	**0.999**	**0.999**	**0.998**	**0.999**	**0.999**	**0.999**
Torus	0.916	0.900	0.916	0.820	0.898	**0.996**	**0.998**
Mobius Strip	0.995	0.996	0.997	0.944	0.988	**0.998**	**0.999**
Mocap	0.991	0.782	0.858	0.829	0.914	**0.996**	**0.996**
Can Opener Kinematic Model	0.953	0.351	0.536	0.316	0.517	**0.996**	**0.997**
Bottle Opener Kinematic Model	0.985	0.46	0.686	0.243	0.491	**0.993**	**0.993**

Numerical accuracy (according to the manifold trustworthiness metric [6]) of the embeddings produced by ISOMAP [5], an atlas learning autoencoder [2], and our method. Best results for each experiment are bolded.

Acknowledgements

References

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[5] Gashler, M et al. (2012). *Connection Science* 24.1 pp. 57-69.
[6] Venna, J et al. (2001). *Proc. ICANN* pp. 485-491.